Measuring excess energy from galaxy feedback with mathematical modelling in R

Alastair Sanderson

School of Physics & Astronomy, University of Birmingham, UK

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2012 useR! conference

Outline

Overview

- Astrophysical context
  - energy input from cosmic feedback (black holes, supernovae)
- Mathematical model
  - solving $T(r)$ & $\rho(r)$ structure of hot gas in clusters of galaxies
- Gas cooling
  - exploring the energy impact of gas loss via cooling
- Summary

Generic details

- Mathematical model construction via solution of (coupled) differential equation(s): deSolve package
- Multiple evaluation and capture of results over a grid of models: plyr package
- Visualization of structured data: ggplot2 package
- Good demonstration of R's powerful capabilities in easily combining numerical analysis with structured data handling and visualization
  - $\rightarrow$ rapid prototyping pipeline

Outline

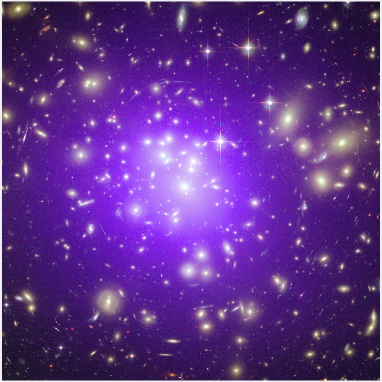
Clusters of galaxies

- Clusters of galaxies are the largest gravitationally bound objects in the Universe; 1–2 Mpc radius ($\sim 5 \times 10^{22}$m):

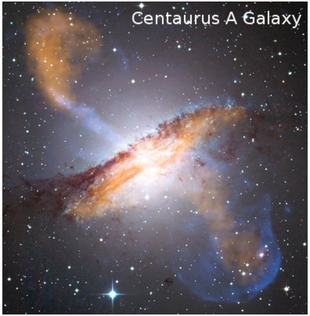
Typical mass composition

- $\sim$ 82% dark matter
- $\sim$ 15% hot gas ($\sim 10^{7-8}$ K, $\rho \sim 10^{-3}$ cm^{-3}) $\rightarrow$ **emits X-rays** (shown in blue)
- $\sim$ 3% stars (mostly in the galaxies)

Abell 1689 galaxy cluster



Cosmic feedback



- The Universe expands & cools → stars form and material pulled into super-massive black holes at the centre of galaxies
  - triggers black hole outbursts (left) and supernova explosions (right) → inject energy into the surrounding gas
  - cooling is (partially) regulated in a *negative feedback cycle*

→ the details of this feedback cycle are poorly understood

Hot gas in galaxy clusters

- Hot gas accounts for majority of normal matter ('baryons') in galaxy clusters
  - retains record of past feedback (non-gravitational heating) in its temperature and density structure
  - gas temperature and density structure can be mapped from its X-ray emission (e.g. Sanderson & Ponman, 2010)
- The gas is approximately in **hydrostatic equilibrium** in the gravitational potential
  - when fluid pressure balances gravity, e.g. within a stable star

Hydrostatic equilibrium

- Thermal pressure balances gravity:

$$\frac{dP_{\mathrm{gas}}}{dr} = -\frac{GM_{\mathrm{total}}(< r)}{r^2}\,\rho_{\mathrm{gas}}$$

$$P = \rho_{\mathrm{gas}} k_B T$$

- $\Rightarrow$ can recast in terms of $\frac{dT}{dr}$ and $\frac{d\rho_{\mathrm{gas}}}{dr}$
- Ignore self-gravity of gas; dark matter dominates total mass
- Can solve this differential equation using the *ode* function in the deSolve R package

Binding energy & excess energy

- Binding energy is simply the energy needed to 'unbind' a gas particle (i.e. move it an infinite distance away):

$$BE = gravitational\ energy - thermal\ energy$$

- Any extra energy, injected by cosmic feedback would *lower* BE, by reducing the extra energy needed for unbinding
- But. . . need a suitable baseline reference model for comparison
  - must compare observed & baseline model BE within the same enclosed gas mass (**not radius!**)

Outline

Simple, self-similar model

- Gravity is a scale-free force: big things should resemble scaled up versions of small things $\rightarrow$ **self-similarity**
- But, 'baryon' physics (i.e. cosmic feedback) breaks symmetry: introduces characteristic scale(s); *smaller things affected more*

- The self-similar baseline model assumes that gas traces mass: i.e. **constant gas fraction**

$$f_{gas} = \frac{M_{\rm gas}(< r)}{M_{\rm total}(< r)} = 0.13$$

- 0.13 is roughly 90% of the gas fraction of the whole Universe

Thermodynamic structure of self-similar model

Need to re-arrange a mass of gas ($M_{\rm gas}$) in the same gravitational potential, with a constant gas fraction, $f_{\rm gas} = 13\%$

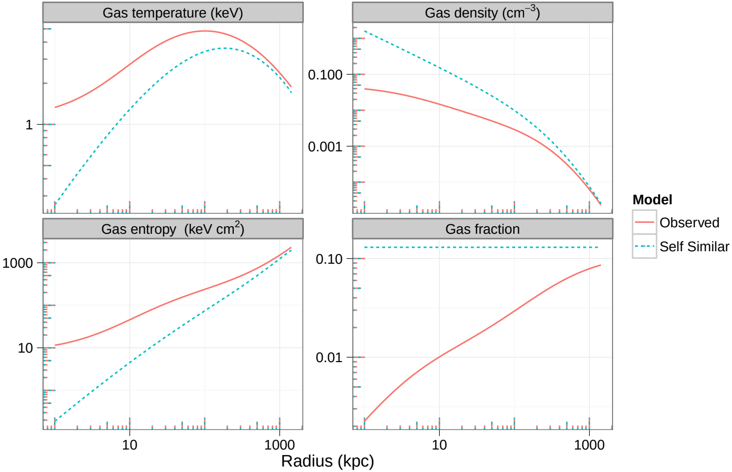
$$M_{\rm gas} = f_{\rm gas} \times M_{\rm tot}$$

$$\rho_{\rm gas}(r) = f_{\rm gas} \times \rho_{\rm total}(r)$$

Assume an **outer** boundary condition of $\frac{d\ln K}{d\ln r} = 1.1$, motivated by theoretical models of galaxy cluster formation (e.g. Tozzi & Norman, 2001)

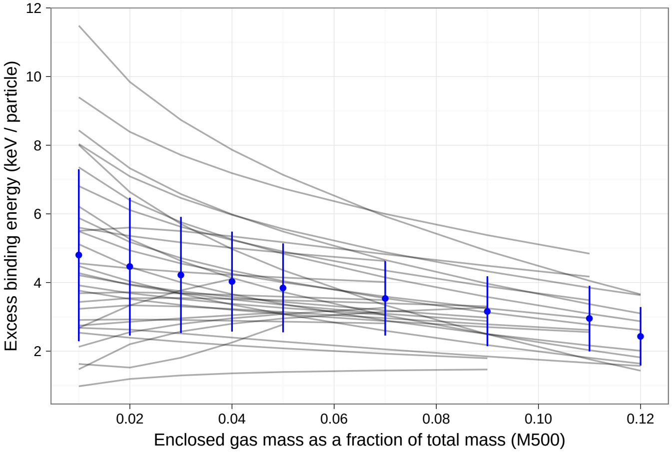
$$\text{‘entropy’},\ K = \frac{k_B T}{\rho_{\rm gas}^{\frac{2}{3}}}$$

Self-similar vs. observed model



- Observed model gas is hotter & less dense → higher entropy

Spatial variation in excess energy



- Radius increases to the right; 1 curve per cluster; self-similar ref. model
- Suggests a centrally concentrated heating source

Outline

Cooling vs. heating

- Gas cools & condenses to (mostly) form stars
  - confined to inner regions, where gas entropy is lowest
  - gas flows in from larger radii to replace it
- Self-similar model ignores cooling: will overestimate the observed excess energy
- Model generalized to allow loss of gas due to cooling
  - gas mass distribution is 'truncated' by a given amount
  - remaining gas rearranged in hydrostatic equilibrium

Truncated gas mass model

- Convective stability $\Rightarrow$ entropy is monotone function of radius
  - $K(M_{\mathrm{gas}})$ fully specifies gas in hydrostatic equilibrium, for a given gravitational potential

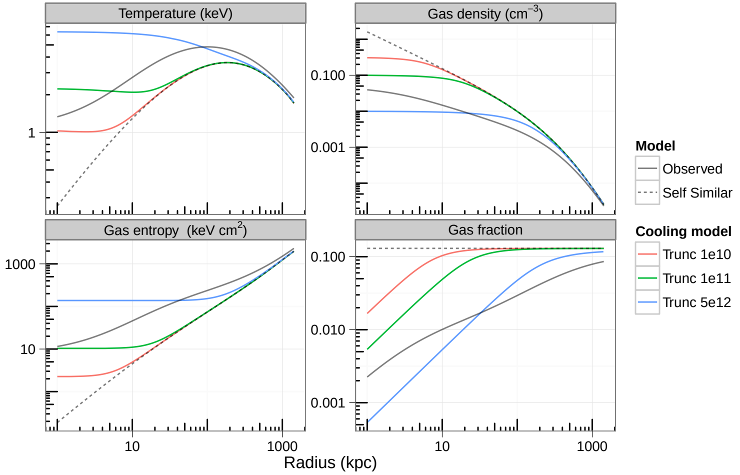
- Truncate $K(M_{\mathrm{gas}})$ by some mass of gas (assumed cooled out)
  - $\rightarrow$ defines new central entropy level: **inner** boundary condition
  - truncated gas mass redistributed within the same potential

- Solve coupled differential equations (Voit et al., 2002):

$$\frac{dP_{\mathrm{gas}}}{dr} = -\frac{GM_{\mathrm{total}}(< r)}{r^2}\,\rho_{\mathrm{gas}} \tag{1}$$

$$\frac{dM_{\mathrm{gas}}}{dr} = 4\pi r^2\,\rho_{\mathrm{gas}} \tag{2}$$

Cooling (truncated) vs. self-similar model



- Lowest entropy gas is removed (would form stars); remaining gas rearranged in hydrostatic equilibrium: ends up hotter → **mimics heating**

Effect of cooling on binding energy

- Entropy is a monotone function of radius $\Rightarrow$ cooling (truncation) removes the most tightly bound gas
  - *binding energy of remaining gas is lower*

Gas mass truncated (Solar)	Excess energy (keV/particle)
- (Self-similar)	1.96
1e10	1.96
1e11	1.92
5e12	1.05

- Excess binding energy of this observed galaxy cluster vs. truncation of 5×10^{12} Solar masses is almost halved
  - 'correct' truncation level needs to be based on the stellar mass within each cluster $\rightarrow$ future work

Outline

Conclusions

- Measured mean **excess** binding energy of gas in galaxy clusters compared to baseline 'Self similar' model & explored effect of cooling, by solving (coupled) differential equation(s)
- Initial results favour centrally concentrated heating source: Active Galactic Nucleus (AGN) in central brightest galaxy, powered by super-massive black hole ($\sim 10^9$ Solar mass)
- Further development of the model needed to understand the impact of both cooling and the epoch of heating
- *R enables smooth integration of mathematical modelling with structured data manipulation and visualization:* deSolve, plyr & ggplot2 packages

Contact details

Alastair Sanderson: http://www.sr.bham.ac.uk/~ajrs
ajrs@star.sr.bham.ac.uk

Birmingham R User Meeting (BRUM) (co-organizer)

www.birminghamR.org 

Acknowledgements & references

These slides were written as a beamer presentation in (Emacs) org mode

Image credits

- Abell 1689 galaxy cluster (X-ray: NASA/CXC/MIT/E.-H Peng et al; Optical: NASA/STScI)
- Centaurus A galaxy (X-ray: NASA/CXC/CfA/R.Kraft et al.; Submillimeter: MPIfR/ESO/APEX/A.Weiss et al.; Optical: ESO/WFI)
- M82 galaxy (X-ray: NASA/CXC/JHU/D.Strickland; Optical: NASA/ESA/STScI/AURA/The Hubble Heritage Team; IR: NASA/JPL-Caltech/Univ. of AZ/C. Engelbracht)

References

- Sanderson & Ponman, 2010, Mon. Not. of Royal Astr. Soc., 402, 65–72 (ADS | DOI)
- Tozzi & Norman, 2001, Astrophysical Journal, 546, 63–84 (ADS | DOI)
- Voit, Bryan, Balogh & Bower, 2002, Astrophysical Journal, 576, 601–624 (ADS | DOI)